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VORTEX-INDUCED VIBRATION OF A FLEXIBLE CANTILEVER

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This study is concerned with the vortex-induced vibrations of a flexible cantilever in a fluid flow. Our cantilever comprises a leaf spring encased within a rubber flexible cylinder, restricting the vibrations of the body in a water channel flow to principally transverse motion. It is found that the transverse amplitude response of the cantilever has a marked similarity with transverse vibrations of an elastically mounted rigid cylinder, in that there is a clear initial branch extending to high amplitudes, with a jump to a lower branch response, as normalized velocity is increased. The continuous initial branch suggests that a distinct upper branch does not exist for the cantilever, as is found for a rigid cylinder under similar conditions of low mass and damping. Good agreement is found between the response amplitude and frequency for two "identical" cantilevers, one set up by Pesce and Fujarra, where strain is measured to infer the body dynamics, and the other arrangement by Flemming and Williamson, where the tip motion is measured using optical techniques. An interesting large-amplitude response mode is found at higher normalized velocities ($U^* > 12$) outside the principal synchronization regime (typically $U^* = 4$–8), which is observed for an increasing velocity, or may be triggered by manual streamwise disturbances of the body. This vibration mode is due to a coupled streamwise-transverse motion, where the streamwise amplitude becomes non-negligible, and may be related to a further vibration mode at high normalized speed, found for a vibrating pivoted rod, by Kitagawa *et al.* (1999).

1. INTRODUCTION

In this work, we are concerned with the vortex-induced vibrations of a flexible cantilever in a fluid flow. This configuration, of some practical significance, has received only little attention in the literature to date. The early studies of Vickery & Watkins (1964) and King (1974) demonstrated large-amplitude tip vibration of around 1·5–1·6 diameters, while recent related studies of cantilever dynamics have been undertaken at São Paulo (Fujarra *et al.* 1998; Pesce & Fujarra 2000), showing comparable tip amplitudes of around 1·7 diameters. The relation between the dynamics of such cantilevers and elastically mounted rigid cylinders has been briefly addressed in Pesce & Fujarra (2000), and it is apparent that the

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amplitude variation, as one increases flow velocity, exhibits a similar discontinuity between two response branches to what is found in elastically mounted rigid cylinder studies. A comparison between the peak amplitudes for cantilevers and rigid cylinders was undertaken by Griffin & Skop (1976). In that study, they attempted to collapse the peak amplitudes versus a mass-damping parameter in what we call a "Griffin plot" (Khalak & Williamson, 1999), although even using an "eigenmode factor", γ (where $\gamma = 1{\cdot}305$), to normalize the cantilever amplitude, the peak values do not collapse well. In a further relevant study, the dynamics of pivoted rods has been investigated by Kitagawa *et al.* (1999), who observe a vibration mode at high speed, outside the principal synchronization regime, which they attribute to the influence of an end cell of lower frequency vortex shedding near the tip of the rod. A more recent study of vibration of pivoted rods has been presented in a seminar by Atsavapranee & Wei (1999), who find large-amplitude oscillations comparable to the present cantilever dynamics.

In the present investigation, the cylindrical cantilever is constructed of an elastomeric material, but within this structure is a thin flexible aluminium plate, allowing much more flexibility in the transverse direction to the fluid flow, than exists in the streamwise direction. In the case of a cantilever whose flexibility is the same in the transverse and streamwise directions (Pesce & Fujarra 2000), there are distinct similarities of the response with the elastically mounted rigid cylinder case (Khalak & Williamson 1997*a*), as shown in Figure 1. The cantilever study was chosen to have a similar mass (m^*) and damping (ζ) to the rigid cylinder arrangement [$m^* = $ (oscillating mass)/(displaced fluid mass) $=$ 2·4; leading to a mass-damping parameter $m^*\zeta = 0{\cdot}016$, where $\zeta =$ structural damping ratio]. Although the rigid cylinder studies of Khalak & Williamson (1996, 1997*b*, 1999) show clearly a three-branch type of response, comprising an initial, upper, and lower branch of amplitude response, it appears that the response of the cantilever in Figure 1 exhibits a single "initial" branch, which then drops to a lower branch; in essence showing only a two-branch type of response, despite the very low mass and damping. The case of the flexible cantilever cannot be considered equivalent to the elastically restrained rigid cylinder, because the amplitude varies along the span in the case of the cantilever. Nevertheless, the response amplitude plots in Figure 1 are surprisingly similar in overall shape, especially along the lower segment of the initial response branch, and along the lower branch. This agreement is markedly improved, as in Figure 1, if one dispenses with the classical use of the "eigenmode factor", mentioned above, which has been employed in past work to normalize cantilever amplitudes.

An interesting point in the cantilever studies of Pesce & Fujarra (2000) is the existence, at the highest oscillation amplitudes (at the top of the left-hand response branch in Figure 1), of coupled streamwise-transverse oscillations. It was felt that by using a leaf spring in the present investigation, these stream-wise oscillations and their coupling with the transverse vibrations, could be largely prohibited. While this might be true for the principal synchronization regime ($U^* = 4$–12), where the streamwise oscillations are negligible, we find instead that such a coupling can still exist at higher speeds. For the present cantilever, the stiffness and natural frequency are greater in the streamwise direction, which therefore shifts the regime of coupling to higher normalized flow speeds. What might be found surprising is the large magnitude of the oscillation amplitudes due to this coupling, and these are discussed later, with reference to Figure 4.

2. EXPERIMENTAL DETAILS

The cantilever in the present work is constructed of rubber, but within this cylindrical material is a thin flexible aluminium plate, as discussed earlier. The diameter of the cylinder

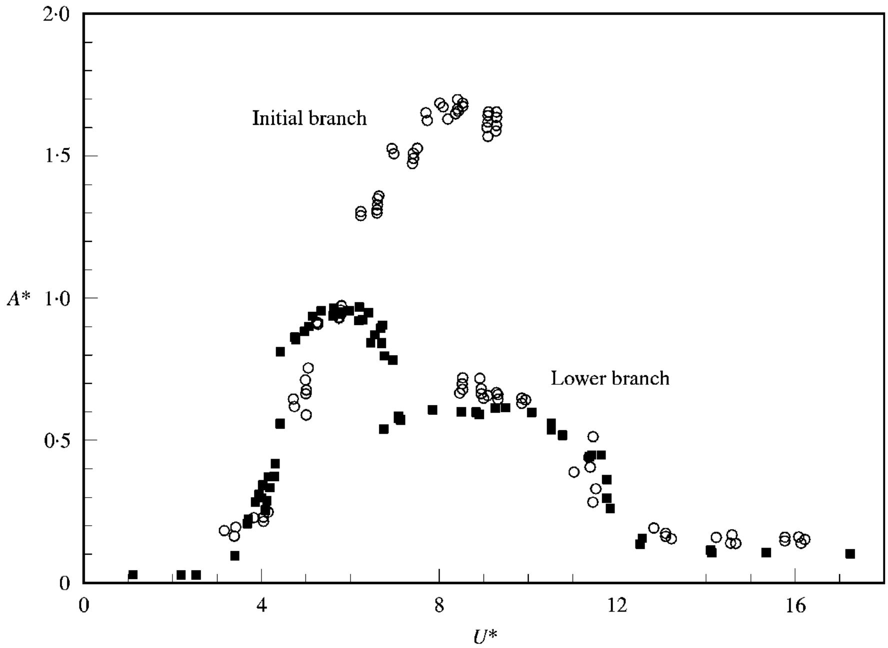

Figure 1. Comparison between amplitude response (A^*) for a flexible cantilever and an elastically mounted rigid cylinder, as a function of normalized velocity, U^*. The cantilever in this case has the same stiffness in the streamwise and transverse directions. ○ Flexible cantilever; ■ Rigid cylinder.

is 10 mm, giving an immersed length–diameter ratio of 41. Two "twin" cantilevers have been constructed expertly at University of São Paulo, and subsequently placed in two different water channel facilities, one in São Paulo (and at University of Michigan during the sabbatical of CPP and ALCF in 1999), and one at Cornell. The ratio of stiffness between the streamwise-transverse oscillations is 18·9. Both of the cylinders have a mass ratio, $m^* = 1{\cdot}3$ and a mass-damping $(m^* + C_A)\zeta = 0{\cdot}185$ and are identically clamped. The blockage ratios are 2·6% (Cornell) and 1% (Michigan). The free-stream turbulence is less than 0·9% in both facilities. Over the range of Reynolds numbers, Re = 1000–2500, we take the Strouhal number as 0·208. The "Brazilian" cantilever is arranged with strain gauges, to infer the tip amplitudes. The tip amplitudes for the "Cornell" cantilever are measured directly using an optical bi-axial displacement transducer. In both arrangements, the initially vertical cantilever is clamped just above the water surface, and the gap between the cantilever tip and a false end plate within the channel, is kept close to 1 mm.

3. DYNAMICS OF THE FLEXIBLE CANTILEVER

Corresponding with the recent work in Pesce & Fujarra (2000), the response amplitude ($A^* = A/D$), as a function of normalized velocity (U^*) seems to exhibit two distinct branches, labelled here as the "initial" branch and the "lower" branch, and shown in Figure 2. (We define the normalized velocity by $U^* = U/f_n D$, with U = free-stream velocity, f_n = the natural frequency in water, D = diameter). Comparisons between techniques to measure amplitude response (computing the tip amplitude from strain data, versus using direct measurement of the tip deflections using optical methods) are quite reasonable, as shown in Figure 2. Despite the differences in the amplitudes of the lower branch, the oscillation frequencies match very well, and show similar behaviour to what is found for

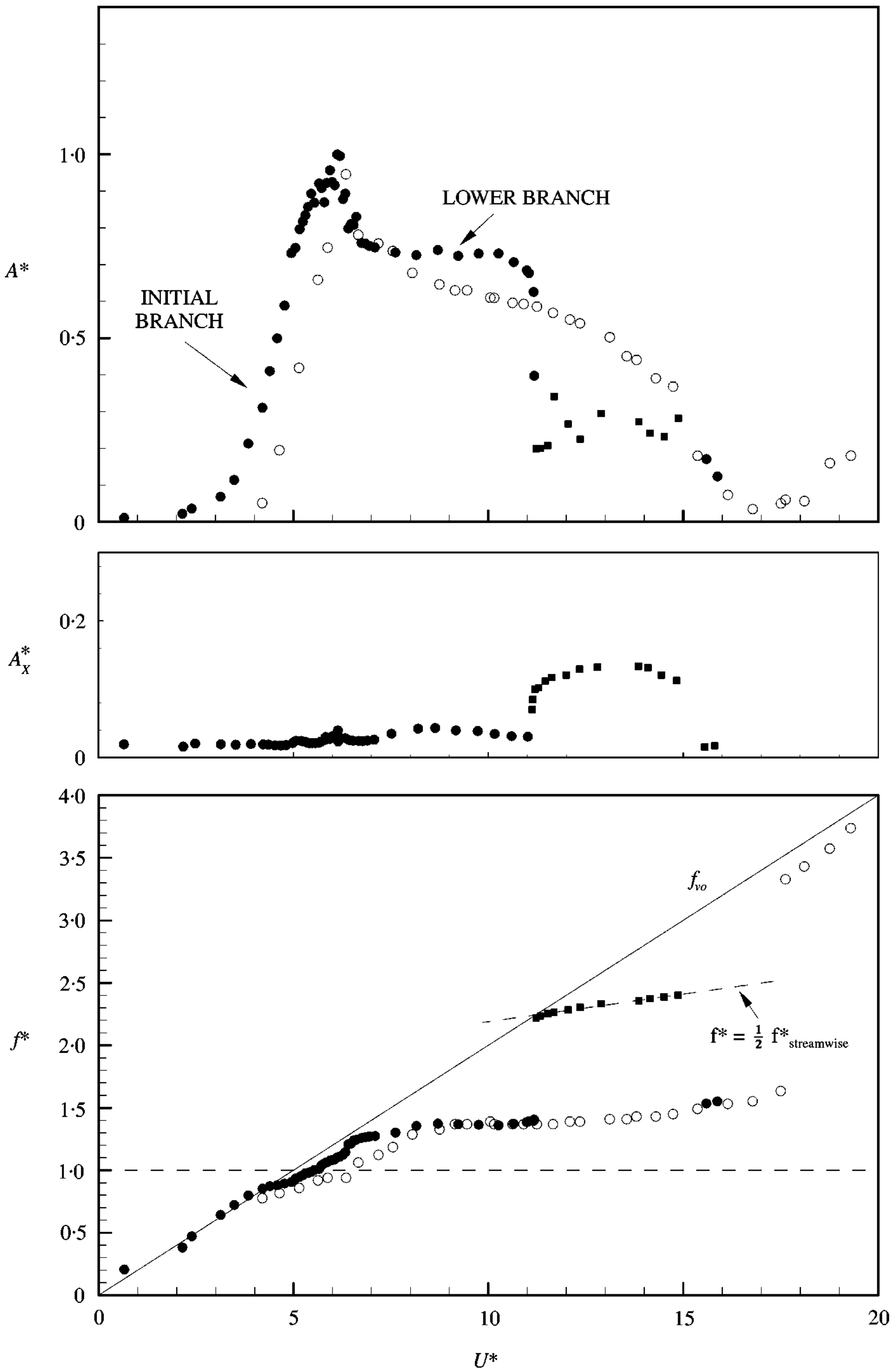

Figure 2. Amplitude response in the transverse direction (A^*) and in the streamwise direction (A_x^*), and frequency response (f^* = oscillation frequency/natural frequency), versus normalized velocity, U^*. The streamwise amplitudes were measured simultaneous with transverse vibrations, using an optical transducer (in the case of the solid symbols). ○, cantilever (strain); ●, cantilever (optical); ■, cantilever (optical) where comparable streamwise oscillations exist.

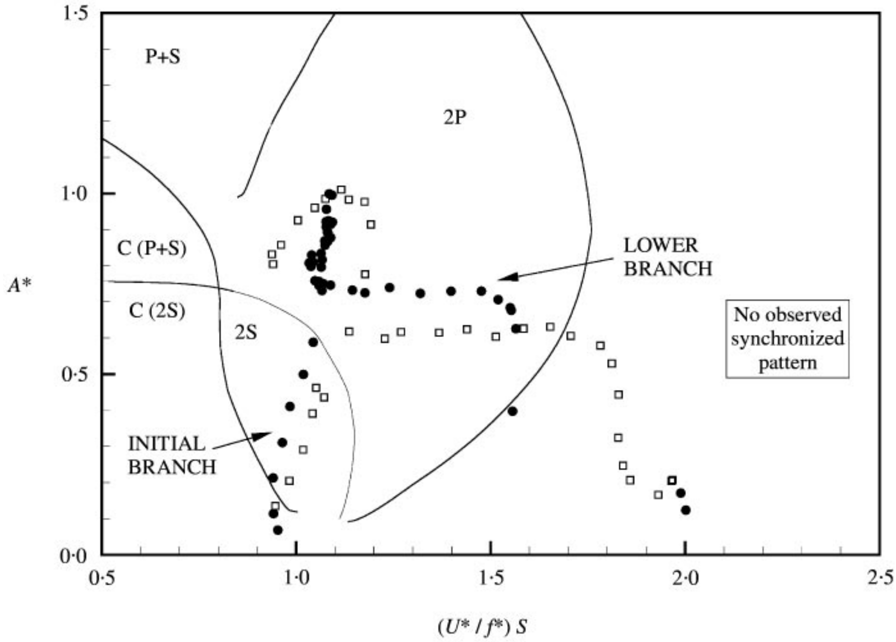

Figure 3. Comparison of cantilever amplitude response (●) with the response of an elastically-mounted rigid cylinder (□) at similar mass ratios ($m^* = 1{\cdot}3$ and $1{\cdot}2$, respectively), using the "true normalized velocity", $(U^*/f^*)S$. These amplitudes are superposed onto a map of vortex formation modes (2S, 2P, etc) as defined in Williamson & Roshko (1988).

rigid-cylinder vibrations. They lie well above the natural frequency (for $U^* > 6$), and this is a characteristic of low mass ratio vortex-induced vibrations, as shown for example in Khalak & Williamson (1997*b*) and Gharib *et al.* (1998). In the presentation of Figure 2, the "initial" branch appears to be a single continuous branch.

It has recently been clarified by Govardhan & Williamson (2000) that elastically mounted rigid cylinders exhibit a three-mode response (initial-upper-lower branches) for low mass-damping ($m^*\zeta$), and a two-mode response (only initial-lower branches) for high mass-damping. If we now directly compare the cantilever data, in Figure 3, with free vibrations of a rigid cylinder (Govardhan & Williamson 2000), at similar mass ratios ($m^* = 1{\cdot}3$ and $1{\cdot}2$, respectively), they both exhibit an initial branch and a lower branch. However, the rigid cylinder case exhibits the three-mode type of response, and it is known that the data above $A^* = 0{\cdot}6$ corresponds to a separate "upper" branch, with a distinct vortex formation mode. The data in Figure 3 indicate that possibly the cantilever also has an "upper" branch, although to prove this point one would need to demonstrate a discontinuity in the initial response branch. Such a discontinuity is quite clear in the case of the rigid cylinder. In the case of the cantilever, it is possible that two vortex formation modes, defined as 2P and 2S mode (Williamson & Roshko 1988) exist simultaneously along the span, as suggested by the mode boundaries in Figure 3. Such a "hybrid" mode was found by Techet *et al.* (1998) for an oscillating tapered cylinder. Further understanding of this point would be forthcoming with the implementation of the PIV technique on this problem, and such experiments are presently planned.

A further interesting result for the cantilever is the large-amplitude transverse vibration response for high speeds, $U^* > 12$, in Figure 4. (These results are from the cantilever fitted with strain gauges, and comprise a separate set of experiments to those in Figure 2. We also

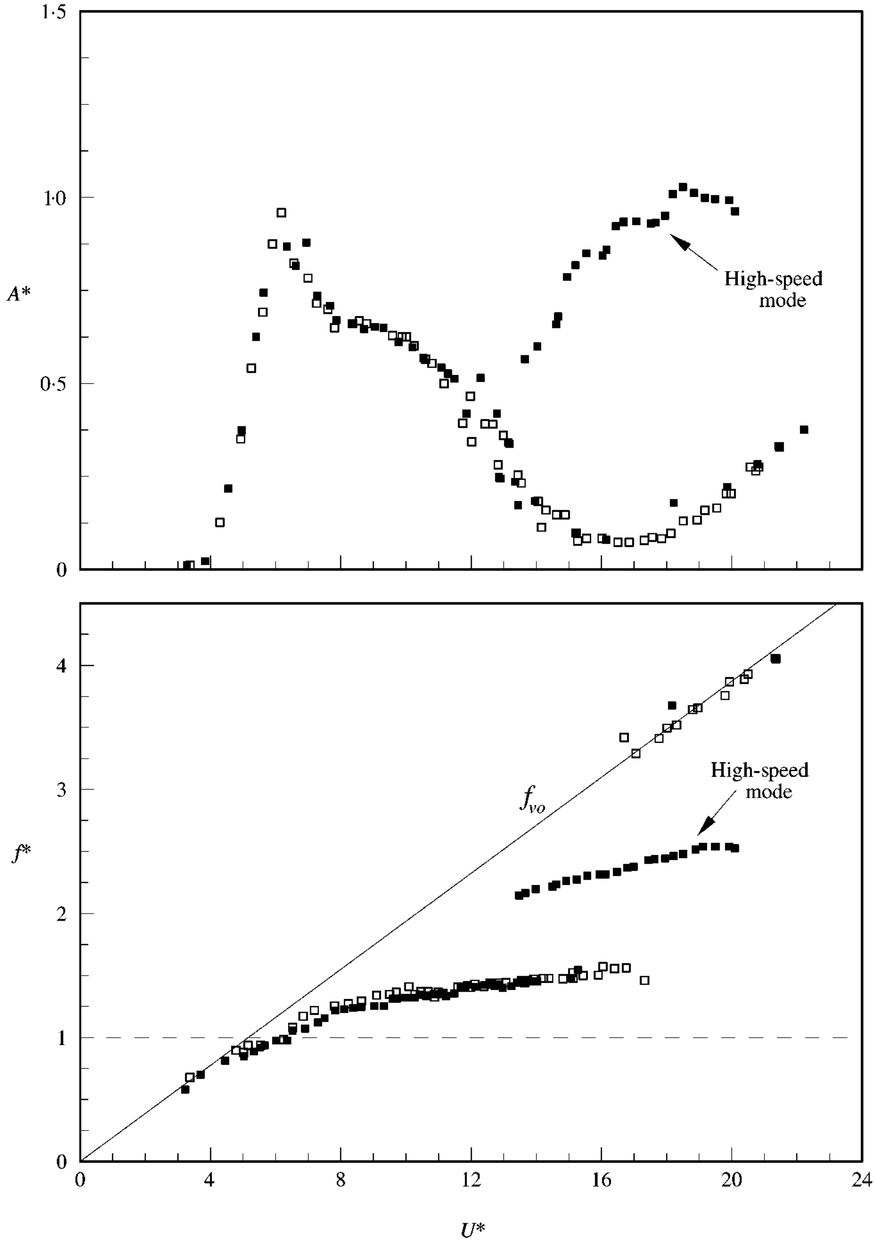

Figure 4. Tip amplitude and frequency response versus normalized velocity, as measured from the cantilever instrumented with strain gauges: ■, increasing velocity; □, decreasing velocity.

include here the upper amplitudes for $U^* > 12$, which were omitted from Figure 2, for clarity.) These data, along the high-amplitude branch for $U^* > 12$, are for increasing velocity, though evidence that the high-amplitude branch can be reached for decreasing velocities will be shown in a further more comprehensive study (Fujarra *et al.* 2001). The cause of this mode is apparently related to the simultaneous presence of streamwise and transverse vibrations. (This might be expected, because the natural frequency in the streamwise direction is roughly 4 times that for the transverse direction, yielding an expected streamwise response at around $U^* \sim 20$.) In the other set of experiments (using optical diagnostics), shown in Figure 2, the square symbols for $U^* > 12$ at amplitudes $A^* \sim 0{\cdot}25$, correspond to conditions where the streamwise and transverse vibrations are comparable. There is evidently a strong correlation between the streamwise and transverse vibrations, as we find that the frequency of transverse vibration (square symbols in Figure 2) is exactly half the frequency for the streamwise oscillations, as measured using the optical technique. One may deduce that the frequency of the high-speed response of Figure 4 also reflects a streamwise-transverse coupling, and in fact this is clear from visual observation in the experiments, where the cantilever tip follows a figure-of-eight trajectory.

A similar response mode has been found by Kitagawa *et al.* (1999), at high normalized velocities, $U^* = 14$–17, which they attribute to the influence of forcing from a vortex shedding cell of low frequency adjacent to the tip of their pivoted rod. It is conceivable that their response mode could also be associated with a coupled streamwise-transverse motion as found in the present work, although their oscillation amplitudes are far smaller, $A^* \approx 0{\cdot}05$, and no streamwise amplitudes were measured to investigate this point.

4. CONCLUSIONS

In summary, vortex-induced vibrations of a cantilever exhibit distinctly similar response modes as found for the elastically restrained rigid cylinder, despite the fact that the cantilever oscillation amplitude varies along the span. As velocity is increased, it appears that there are two branches of response in the case of the cantilever; whereas in the case of the elastically mounted rigid cylinder, three response branches are found, at comparable values of low mass and damping. Evidence for the existence of three response branches for the cantilever (not only an initial and lower branch, but also a separate upper branch) would require demonstration of a discontinuity in the initial response branch. It is planned to investigate the wake vortex dynamics at different points along the span, using the DPIV technique to determine wake vorticity.

A high-speed mode of large-amplitude response has been found, which is outside the principal synchronization regime, and which is associated with a streamwise-transverse vibration coupling. The natural frequency of the cantilever is higher in the streamwise direction (it has been made stiffer in that direction by enclosing a leaf spring within the rubber cantilever), and so the streamwise oscillations are excited at higher speeds. When streamwise vibrations are stimulated, the transverse frequency of this high-speed mode corresponds to precisely half the streamwise vibration frequency. Although the high-speed vibration mode of a pivoted rod, investigated by Kitagawa *et al.* (1999), yields much smaller amplitudes, it is conceivable that their vibrations may be related to coupled streamwise-transverse oscillations of the type observed here, since they also used a leaf spring as their pivot, restricting most of the motion to the transverse direction.

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